

# Scale-invariant spectrum of Lee-Wick model in de Sitter spacetime

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## Abstract

We obtain a scale-invariant spectrum from the Lee-Wick model in de Sitter spacetime. This model is a fourth-order scalar theory whose mass parameter is determined by  $M^2 = 2H^2$ . The Harrison-Zel'dovich scale-invariant spectrum is obtained by Fourier transforming the propagator in position space as well as by computing the power spectrum directly. It shows clearly that the LW scalar theory provides a truly scale-invariant spectrum in whole de Sitter, while the massless scalar propagation in de Sitter shows a scale-invariant spectrum in the superhorizon region only.

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# 1 Introduction

The Lee-Wick (LW) model [1, 2] of a fourth-order derivative scalar theory with  $\phi$  has provided a cosmological bounce which could avoid the singularity and give a scale-invariant spectrum [3]. Introducing an auxiliary field (LW scalar) and a normal scalar  $= \phi + \text{LW scalar}$  [4], the fourth-order Lagrangian can be expressed in terms of two second-order Lagrangians. Here the kinetic and mass terms of the LW scalar have the opposite sign when compared with those for the normal scalar. Since the LW scalar is a ghost scalar, it provides a bouncing solution. Perturbations of normal scalar generated in the contracting phase have survived during bouncing and have led to a scale-invariant spectrum in the expanding phase [5, 6]. Thus, the LW model is considered as a possible alternative to inflationary cosmology.

It is well-known that the power spectrum of a massless minimally coupled scalar (mmc) in de Sitter (dS) spacetime takes the form of  $(H/2\pi)^2[1 + k^2/(a^2H^2)]$  which reduces to the Harrison-Zel'dovich (HZ) scale-invariant spectrum of  $(H/2\pi)^2$  in the superhorizon region of  $k \ll aH$  [7]. On the other hand, the quantization of a mmc scalar field in dS spacetime has a nontrivial problem due to the appearance of IR divergence in compared to a massive minimally coupled scalar [8]. They then propose to trade dS SO(1,4) invariance for a smaller SO(4) invariance. Actually, the construction of the mmc scalar Green function in coordinate-space has remained a matter of controversy and has been considered as a subject of debate for past decades. After subtracting the divergent term, however, one has gotten a renormalized Green function (propagator)  $G_{\text{mmc}}(Z(x, x'))$  with a tractable drawback [9, 10] where  $Z(x, x')$  is dS SO(1,4) invariant distance. Using Cesaro-summation method to tame a divergent Fourier transform properly, Youssef has recently recovered the original form of power spectrum but not a scale-invariant (equal amplitude on all scales) spectrum in whole dS region [11].

At this stage, we ask an important question “what kind of a scalar model could provide a truly scale-invariant power spectrum in whole dS evolution”. The answer is that it would be the LW model with mass parameter  $M^2 = 2H^2$ . In this case, we will not introduce the auxiliary field method (normal and LW scalars to lower the LW model to a second-order theory with two scalars). Instead, one uses the Ostrogradski formalism [12, 13] and their equivalence was proved in appendix B of Ref.[14]. The LW operator  $\Delta_4 = -\bar{\nabla}^2(-\bar{\nabla}^2 + 2H^2)$  is a conformally covariant fourth-order operator since it transforms  $\Delta_4 = e^{-4\sigma}\tilde{\Delta}_4$  under the

conformal transformation of  $g_{\mu\nu} \rightarrow e^{2\sigma} \tilde{g}_{\mu\nu}$  in dS spacetime [15]. Furthermore, its propagator takes the form  $\tilde{D}(Z(x, x')) = [G_{\text{mmc}}(Z(x, x')) - G_{\text{mcc}}(Z(x, x'))] = -\frac{H^2}{8\pi^2} \ln[1 - Z(x, x')]$  where  $G_{\text{mcc}}(Z(x, x'))$  represents the propagator of a massless conformally coupled (mcc) scalar. It turns out that taking Cesaro-summation method to define a divergent Fourier transform of  $\tilde{D}(Z(x, x'))$  leads to an exactly scale-invariant spectrum of  $(H/2\pi)^2$  without scale-dependence  $k$ . This implies that the HZ scale-invariant spectrum corresponds to a logarithmic zero conformal weight distribution in the coordinate-space  $S^2$  angular directions on the sky [10]. In this case,  $Z(x, x') = \hat{n} \cdot \hat{n}'$  is the cosine of the angle between the two direction vectors on the sky viewed from the origin where the radiation appears to originate.

In this work, we compute the power spectrum of the LW model directly by employing the quantization scheme of Pais-Uhlenbeck fourth-order oscillator [12] and taking the Bunch-Davies vacuum. As was expected, we obtain the same scale-invariant spectrum of  $(H/2\pi)^2$ . This implies clearly that a scale-invariant spectrum preserves dS  $SO(1,4)$  invariance.

## 2 Einstein-Lee-Wick gravity

Let us first consider the Einstein-Lee-Wick (ELW) gravity whose action is given by

$$S_{\text{ELW}} = S_{\text{E}} + S_{\text{LW}} = \int d^4x \sqrt{-g} \left[ \left( \frac{R}{2\kappa} - \Lambda \right) + \frac{1}{2} \left( \phi \nabla^2 \phi - \frac{1}{M^2} (\nabla^2 \phi)^2 \right) \right], \quad (1)$$

where  $\kappa = 8\pi G = 1/M_{\text{P}}^2$ ,  $M_{\text{P}}$  being the reduced Planck mass and  $M^2$  is a mass parameter determined to be  $2H^2$ . Here,  $S_{\text{E}}$  denotes the action for the Einstein gravity with positive cosmological constant, whereas  $S_{\text{LW}}$  is the LW scalar action which differs slightly from the LW standard action including a mass term  $m^2 \phi^2$  and potential  $\frac{g}{3!} \phi^3$  [4].

Varying the action (1) with respect to the metric tensor  $g_{\mu\nu}$  leads to the Einstein equation

$$G_{\mu\nu} + \kappa \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \quad (2)$$

where the energy-momentum tensor takes the form

$$\begin{aligned} T_{\mu\nu} = & \nabla_\mu \phi \nabla_\nu \phi + \frac{1}{2} g_{\mu\nu} \nabla_\rho \phi \nabla^\rho \phi \\ & - \frac{1}{M^2} \left[ 2 \nabla_\mu (\nabla^2 \phi) \nabla_\nu \phi + g_{\mu\nu} \nabla_\rho (\nabla^2 \phi) \nabla^\rho \phi - \frac{1}{2} g_{\mu\nu} \nabla^2 \phi \nabla^2 \phi \right]. \end{aligned} \quad (3)$$

On the other hand, the scalar equation for the action (1) is given by

$$\nabla^2 \phi - \frac{1}{M^2} \nabla^2 \nabla^2 \phi = 0 \rightarrow -\frac{1}{M^2} \nabla^2 (\nabla^2 - M^2) \phi = 0. \quad (4)$$

A solution of dS spacetime to Eq.(2) can be easily found when one chooses the vanishing scalar

$$\bar{R} = 4\kappa\Lambda, \quad \bar{\phi} = 0. \quad (5)$$

In this case, the Riemann and Ricci tensors can be written by

$$\bar{R}_{\mu\nu\rho\sigma} = H^2(\bar{g}_{\mu\rho}\bar{g}_{\nu\sigma} - \bar{g}_{\mu\sigma}\bar{g}_{\nu\rho}), \quad \bar{R}_{\mu\nu} = 3H^2\bar{g}_{\mu\nu} \quad (6)$$

with Hubble constant  $H^2 = \kappa\Lambda/3$ . Also, the dS spacetime can be represented by introducing either cosmic time  $t$  or conformal time  $\eta$  as

$$ds_{\text{dS}}^2 = \bar{g}_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j \quad (7)$$

$$= a(\eta)^2[-d\eta^2 + \delta_{ij}dx^i dx^j], \quad (8)$$

where  $a(t)$  and  $a(\eta)$  are cosmic and conformal scale factors expressed by

$$a(t) = e^{Ht}, \quad a(\eta) = -\frac{1}{H\eta}. \quad (9)$$

During the de Sitter stage,  $a$  goes from small to a very large value like  $a_f/a_i \simeq 10^{30}$  which implies that the conformal time  $\eta = -1/aH(z = -k\eta)$  runs from  $-\infty(\infty)$ [the infinite past] to  $0^-(0)$  [the infinite future]. The dS SO(1,4) invariant distance between two spacetime points  $x^\mu$  and  $x'^\mu$  is given by

$$Z(x, x') = \frac{1}{2} \left[ 1 - \frac{H^2 e^{H(t+t')}}{2} |\mathbf{x} - \mathbf{x}'|^2 + \cosh[H(t - t')] \right], \quad (10)$$

$$Z(x, x') = 1 - \frac{1}{4\eta\eta'} \left[ -(\eta - \eta')^2 + |\mathbf{x} - \mathbf{x}'|^2 \right], \quad (11)$$

where the former is the distance when using (7), while the latter is the distance for (8).

### 3 Scalar propagation in dS spacetime

To investigate the cosmological perturbation around the dS spacetime (8), we might choose the Newtonian gauge as  $B = E = 0$  and  $\bar{E}_i = 0$ . Under this gauge, the corresponding perturbed metric with transverse-traceless tensor  $\partial_i h^{ij} = h = 0$  and perturbed scalar can be written as

$$ds^2 = a(\eta)^2 \left[ -(1 + 2\Psi)d\eta^2 + 2\Psi_i d\eta dx^i + \left\{ (1 + 2\Phi)\delta_{ij} + h_{ij} \right\} dx^i dx^j \right], \quad (12)$$

$$\phi = \bar{\phi} + \varphi. \quad (13)$$

Now we linearize the Einstein equation (2) around the dS background to obtain the cosmological perturbed equations. It is known that the tensor perturbation is decoupled from scalars and its equation becomes

$$\delta R_{\mu\nu}(h) - 3H^2 h_{\mu\nu} = 0 \rightarrow \bar{\nabla}^2 h_{ij} = 0. \quad (14)$$

We mention briefly how do two scalars  $\Psi$  and  $\Phi$ , and a vector  $\Psi_i$  go on. The linearized Einstein equation requires  $\Psi = -\Phi$  which was used to define the comoving curvature perturbation in the slow-roll inflation and thus, they are not physically propagating modes. During the dS inflation, no coupling between  $\{\Psi, \Phi\}$  and  $\varphi$  occurs because of  $\bar{\phi} = 0$ . Lastly, the vector is also a non-propagating mode in the ELW theory because it has no kinetic term.

On the other hand, the linearized scalar equation is given by

$$\bar{\nabla}^2(\bar{\nabla}^2 - M^2)\varphi = 0. \quad (15)$$

Hereafter we choose  $M^2$  to be  $2H^2$  to get a msc scalar sector.

In order to find the solution to the linearized fourth-order equation (15) in the whole range of  $\eta$ , we decompose (15) into two second-order equations

$$\bar{\nabla}^2 \varphi^{(\text{mmc})} = 0, \quad (16)$$

$$(\bar{\nabla}^2 - 2H^2)\varphi^{(\text{mcc})} = 0, \quad (17)$$

where  $\varphi = \varphi^{(\text{mmc})} + \varphi^{(\text{mcc})} \equiv \varphi^{(1)} + \varphi^{(2)}$ . This is always possible to occur for a direct product form of fourth-order equation as in (15). Expanding  $\varphi^{(i)}$  in terms of Fourier modes  $\phi_{\mathbf{k}}^{(i)}(\eta)$

$$\varphi^{(i)}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \phi_{\mathbf{k}}^{(i)}(\eta) e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (18)$$

equations (16) and (17) become

$$\left[ \frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 \right] \phi_{\mathbf{k}}^{(1)} = 0, \quad (19)$$

$$\left[ \frac{d^2}{dz^2} - \frac{2}{z} \frac{d}{dz} + 1 + \frac{2}{z^2} \right] \phi_{\mathbf{k}}^{(2)} = 0 \quad (20)$$

with  $z = -\eta k$ . Solutions to (19) and (20) are easily found to be

$$\phi_{\mathbf{k}}^{(1)} = \mathcal{C}_1(i + z)e^{iz}, \quad (21)$$

$$\phi_{\mathbf{k}}^{(2)} = \mathcal{C}_2 i z e^{iz}, \quad (22)$$

where  $\mathcal{C}_1$  and  $\mathcal{C}_2$  are constants to be determined.

## 4 Propagator in de Sitter

We wish to find the power spectrum of perturbed scalar by making Fourier transform of propagator in dS spacetime. First of all, we introduce the LW operator defined by [15, 16]

$$\begin{aligned}\Delta_4 &= \bar{\nabla}^4 + 2R^{\mu\nu}\bar{\nabla}_\mu\bar{\nabla}_\nu - \frac{2}{3}R\bar{\nabla}^2 + \frac{1}{3}(\bar{\nabla}^\mu R)\bar{\nabla}_\mu \\ &\xrightarrow{\text{dS}} -\bar{\nabla}^2(-\bar{\nabla}^2 + 2H^2)\end{aligned}\quad (23)$$

which is a conformally covariant fourth-order operator because it transforms  $\Delta_4 = e^{-4\sigma}\tilde{\Delta}_4$  under a rescaling of metric  $g_{\mu\nu} \rightarrow e^{2\sigma}\tilde{g}_{\mu\nu}$  in dS spacetime. Accordingly,  $\sqrt{-g}\Delta_4$  is a conformally invariant operator. The propagator is given by the inverse of  $\Delta_4$  as [10]

$$D(Z(x, x')) = \frac{1}{2H^2} \left[ \frac{1}{-\bar{\nabla}^2} - \frac{1}{-\bar{\nabla}^2 + 2H^2} \right] = \frac{1}{2H^2} [G_1(Z(x, x')) - G_2(Z(x, x'))] \quad (24)$$

where the propagators of 1(mmc) and 2(mcc) scalar in dS spacetime are given by

$$G_1(Z(x, x')) = \frac{H^2}{(4\pi)^2} \left[ \frac{1}{1-Z} - 2\ln(1-Z) + c_0 \right], \quad G_2(Z(x, x')) = \frac{H^2}{(4\pi)^2} \frac{1}{1-Z}. \quad (25)$$

On the other hand, upon choosing the Bunch-Davies vacuum, the propagator of a massive minimally coupled scalar is given by the hypergeometric function [17]

$$G_0(Z, m^2) = \frac{H^2}{(4\pi)^2} \Gamma(\Delta_+) \Gamma(\Delta_-) {}_2F_1(\Delta_+, \Delta_-, 2; Z(x, x')) \quad (26)$$

with  $\Delta_\pm = \frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$  for  $0 < m^2 \leq \frac{9}{4}H^2$ . For a mmc ( $m^2 = 0, \Delta_+ = 3, \Delta_- = 0$ ) scalar, the quantization of a mmc scalar field in dS spacetime has a nontrivial problem due to the appearance of IR divergence ( $\Gamma(0)$ ) in compared to a massive minimally coupled scalar [8]. After subtracting the divergent term, one got a renormalized propagator  $G_1(Z(x, x'))$  in (25) with a tractable drawback [9]. In the case of a mcc scalar with  $m^2 = 2H^2 (\Delta_+ = 2, \Delta_- = 1)$ , the corresponding propagator is given by  $G_2(Z(x, x'))$  [18].

Plugging (25) into (24), its propagator takes the form

$$D(Z(x, x')) = \frac{1}{16\pi^2} \left( -\ln[1 - Z(x, x')] + \frac{c_0}{2} \right) \quad (27)$$

which is a pure logarithm up to an arbitrary additive constant  $c_0$ . Since our propagator relation is read off from (4)

$$\tilde{D}(Z(x, x')) = \left[ \frac{1}{-\bar{\nabla}^2} - \frac{1}{-\bar{\nabla}^2 + 2H^2} \right] = [G_1(Z(x, x')) - G_2(Z(x, x'))], \quad (28)$$

it takes the form

$$\tilde{D}(Z(x, x')) = -\frac{H^2}{8\pi^2} \ln[1 - Z(x, x')] \quad (29)$$

with  $c_0 = 0$  for simplicity. The power spectrum is then formally given by

$$\mathcal{P} = \frac{1}{(2\pi)^3} \int d^3r \, 4\pi k^3 \tilde{D}(Z(\mathbf{x}, t; \mathbf{x}', t)) e^{-i\mathbf{k}\cdot\mathbf{r}}, \quad \mathbf{r} = \mathbf{x} - \mathbf{x}'. \quad (30)$$

It turns out that making use of Cesaro-summation method to compute a divergent Fourier transform of  $\tilde{D}(Z(x, x'))$  [11], we obtain an exactly scale-invariant spectrum

$$\mathcal{P} = \left(\frac{H}{2\pi}\right)^2 \quad (31)$$

without scale-dependence  $k$ .

## 5 Power spectra

In order to find power spectrum for a scalar perturbation in the ELW gravity, we rewrite the fourth-order bilinear action  $\delta S_{\text{LW}}$  (1) as the second-order bilinear action by using the Ostrogradski's formalism for scalar [12, 13] and tensor [19, 20] as

$$\begin{aligned} \delta S_{\text{LW}}^2 = \frac{1}{2} \int d^4x \Big[ & -a^2(\alpha^2 + \partial_i \varphi \partial^i \varphi) - \frac{1}{2H^2} \Big( (\alpha')^2 - 2\partial_i \alpha \partial^i \alpha + \partial^2 \varphi \partial^2 \varphi \\ & + 4aH\alpha\alpha' - 4aH\alpha\partial^2 \varphi \Big) + 2\beta(\alpha - \varphi') \Big], \end{aligned} \quad (32)$$

where  $\alpha \equiv \varphi'$  is a new field,  $\partial^2 = \partial_i \partial^i$ , and  $\beta$  is a Lagrange multiplier. Here the prime ( $'$ ) denotes differentiation with respect to  $\eta$ . From (32), we define the conjugate momenta as

$$\pi_\varphi = \frac{1}{2H^2} (\varphi''' - 2\partial^2 \varphi' + 2aH\partial^2 \varphi), \quad \pi_\alpha = -\frac{1}{2H^2} (\varphi'' + 2aH\varphi'). \quad (33)$$

Then, the canonical quantization is accomplished by imposing commutation relations as follows:

$$[\hat{\varphi}(\eta, \mathbf{x}), \hat{\pi}_\varphi(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'), \quad [\hat{\alpha}(\eta, \mathbf{x}), \hat{\pi}_\alpha(\eta, \mathbf{x}')] = i\delta(\mathbf{x} - \mathbf{x}'). \quad (34)$$

The field operator  $\hat{\varphi}$  can be expanded in Fourier modes as

$$\hat{\varphi}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \left[ \left( \hat{a}_{\mathbf{k}} \phi_{\mathbf{k}}^{(1)}(\eta) + \hat{b}_{\mathbf{k}} \phi_{\mathbf{k}}^{(2)}(\eta) \right) e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]. \quad (35)$$

We also obtain the momentum operator  $\hat{\pi}_\varphi$  by substituting (35) into (33)

$$\begin{aligned} \hat{\pi}_\varphi(\eta, \mathbf{x}) = & \frac{1}{(2\pi)^{\frac{3}{2}}} \frac{1}{2H^2} \int d^3k \left[ \left( \hat{a}_{\mathbf{k}} \left\{ \left( \phi_{\mathbf{k}}^{(1)}(\eta) \right)''' + 2k^2 \left( \phi_{\mathbf{k}}^{(1)}(\eta) \right)' - 2aHk^2 \phi_{\mathbf{k}}^{(1)}(\eta) \right\} e^{i\mathbf{k}\cdot\mathbf{x}} \right. \right. \\ & \left. \left. + \hat{b}_{\mathbf{k}} \left\{ \left( \phi_{\mathbf{k}}^{(2)}(\eta) \right)''' + 2k^2 \left( \phi_{\mathbf{k}}^{(2)}(\eta) \right)' - 2aHk^2 \phi_{\mathbf{k}}^{(2)}(\eta) \right\} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right] \right]. \end{aligned} \quad (36)$$

Similarly,  $\hat{\alpha}(\equiv \hat{\varphi}')$  operator and its momentum operator  $\hat{\pi}_\alpha$  (33) can be expressed as

$$\hat{\alpha}(\eta, \mathbf{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int d^3k \left[ \left\{ \hat{a}_{\mathbf{k}} \left( \phi_{\mathbf{k}}^{(1)}(\eta) \right)' + \hat{b}_{\mathbf{k}} \left( \phi_{\mathbf{k}}^{(2)}(\eta) \right)' \right\} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right], \quad (37)$$

$$\begin{aligned} \hat{\pi}_\alpha(\eta, \mathbf{x}) = & -\frac{1}{(2\pi)^{\frac{3}{2}}} \frac{1}{2H^2} \int d^3k \left[ \hat{a}_{\mathbf{k}} \left\{ \left( \phi_{\mathbf{k}}^{(1)}(\eta) \right)'' + 2aH \left( \phi_{\mathbf{k}}^{(1)}(\eta) \right)' \right\} e^{i\mathbf{k}\cdot\mathbf{x}} \right. \\ & \left. + \hat{b}_{\mathbf{k}} \left\{ \left( \phi_{\mathbf{k}}^{(2)}(\eta) \right)'' + 2aH \left( \phi_{\mathbf{k}}^{(2)}(\eta) \right)' \right\} e^{i\mathbf{k}\cdot\mathbf{x}} + \text{h.c.} \right]. \end{aligned} \quad (38)$$

Substituting (35)-(38) into (34) leads to the commutation relations and Wronskian conditions:

$$[\hat{a}_{\mathbf{k}}, \hat{a}_{\mathbf{k}'}^\dagger] = \delta(\mathbf{k} - \mathbf{k}'), \quad [\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^\dagger] = -\delta(\mathbf{k} - \mathbf{k}'), \quad (39)$$

$$\begin{aligned} & \left[ \phi_{\mathbf{k}}^{(1)} \left\{ \left( \phi_{\mathbf{k}}^{*(1)}(\eta) \right)''' + 2k^2 \left( \phi_{\mathbf{k}}^{*(1)}(\eta) \right)' - 2aHk^2 \phi_{\mathbf{k}}^{*(1)}(\eta) \right\} \right. \\ & \left. - \phi_{\mathbf{k}}^{(2)} \left\{ \left( \phi_{\mathbf{k}}^{*(2)}(\eta) \right)''' + 2k^2 \left( \phi_{\mathbf{k}}^{*(2)}(\eta) \right)' - 2aHk^2 \phi_{\mathbf{k}}^{*(2)}(\eta) \right\} \right] - c.c. = i2H^2, \end{aligned} \quad (40)$$

$$\begin{aligned} & \left[ \left( \phi_{\mathbf{k}}^{(1)} \right)' \left\{ \left( \phi_{\mathbf{k}}^{*(1)}(\eta) \right)'' + 2aH \left( \phi_{\mathbf{k}}^{*(1)}(\eta) \right)' \right\} \right. \\ & \left. - \left( \phi_{\mathbf{k}}^{(2)} \right)' \left\{ \left( \phi_{\mathbf{k}}^{*(2)}(\eta) \right)'' + 2aH \left( \phi_{\mathbf{k}}^{*(2)}(\eta) \right)' \right\} \right] - c.c. = -i2H^2. \end{aligned} \quad (41)$$

We note that inspired by quantization of the Pais-Uhlenbeck fourth-order oscillator [12], two mode operators ( $\hat{a}_{\mathbf{k}}, \hat{b}_{\mathbf{k}}$ ) are necessary to take into account of fourth-order scalar theory. The unusual commutator for ( $\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^\dagger$ ) reflects that the LW model contains the ghost state scalar [14]. Before we proceed, we remind the reader that  $\phi_{\mathbf{k}}^{(1)}$  and  $\phi_{\mathbf{k}}^{(2)}$  are given by (21) and (22), respectively. Making use of the Wronskian conditions (40) and (41) determine these solutions completely as

$$\phi_{\mathbf{k}}^{(1)} = \frac{H}{\sqrt{2k^3}} (i + z) e^{iz}, \quad (42)$$

$$\phi_{\mathbf{k}}^{(2)} = \frac{H}{\sqrt{2k^3}} i z e^{iz}, \quad (43)$$



which implies that  $|\mathcal{C}_1|^2 = |\mathcal{C}_2|^2 = H^2/(2k^3)$ . One checks easily that solutions (42) and (43) also satisfy the initial condition when choosing the Bunch-Davies vacuum  $|0\rangle$  in the subhorizon limit ( $z \rightarrow \infty$ ) of Eqs. (19) and (20).

On the other hand, the power spectrum of the scalar is defined by [7]

$$\langle 0 | \hat{\varphi}(\eta, \mathbf{x}) \hat{\varphi}(\eta, \mathbf{x}') | 0 \rangle = \int d^3k \frac{\mathcal{P}_\varphi}{4\pi k^3} e^{i\mathbf{k} \cdot (\mathbf{x} - \mathbf{x}')}, \quad (44)$$

which leads to the HZ scale-invariant spectrum

$$\mathcal{P}_\varphi = \frac{k^3}{2\pi^2} \left( \left| \phi_{\mathbf{k}}^{(1)} \right|^2 - \left| \phi_{\mathbf{k}}^{(2)} \right|^2 \right) \quad (45)$$

$$= \left( \frac{H}{2\pi} \right)^2 \left[ 1 + \frac{k^2}{(aH)^2} - \frac{k^2}{(aH)^2} \right] = \left( \frac{H}{2\pi} \right)^2. \quad (46)$$

Here we used the Bunch-Davies vacuum state by imposing  $\hat{a}_{\mathbf{k}}|0\rangle = 0$  and  $\hat{b}_{\mathbf{k}}|0\rangle = 0$ , and the minus sign  $(-)$  in (45) appears when using the unusual commutation relation for  $(\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}^\dagger)$ .

Finally, by comparing (14) with (16), the tensor power spectrum is given by

$$\mathcal{P}_h = 2 \times \left( \frac{2}{M_{\text{P}}} \right)^2 \times \mathcal{P}_{\varphi^{(1)}} = \frac{2H^2}{\pi^2 M_{\text{P}}^2} \left[ 1 + \frac{k^2}{(aH)^2} \right], \quad (47)$$

where  $\mathcal{P}_{\varphi^{(1)}}$  is the spectrum for the mmc scalar.

## 6 Discussions

First of all, the LW model is regarded as a simple fourth-order scalar theory. In this work, we have derived the Harrison-Zel'dovich scale-invariant spectrum by Fourier transforming the coordinate-space renormalized propagator of the LW model with mass parameter  $M^2 = 2H^2$  in dS spacetime. In deriving it, we have used Cesaro-summation technique.

Also, the same scale-invariant power spectrum have been found directly by employing the quantization scheme for a Pais-Uhlenbeck fourth-order oscillator and taking the Bunch-Davies vacuum for dS spacetime. This shows that the scale-invariant spectrum comes out, while preserving dS  $\text{SO}(1,4)$  symmetry. In this computation, we have used the Ostrogradski's formalism instead of the auxiliary formalism because we want to derive the power spectrum of a single scalar satisfying a fourth-order equation, but not for the normal and LW scalars satisfying second-order equations, respectively.

As was shown in (47), the tensor spectrum is not scale-invariant in whole dS space but it is scale-invariant in the superhorizon region of  $k \ll aH$ . Hence, we have to find the corresponding tensor theory that is similar to the Lee-Wick scalar theory. We propose that it might be the massive conformal gravity with an appropriate choice of mass parameter [21, 22], which will be explored elsewhere.

Consequently, the HZ scale-invariant spectrum of scalar is not given by a massless scalar theory but by LW scalar theory in dS spacetime. This means that the original dS  $SO(1,4)$  symmetry preserves in computing propagator (power spectrum) of LW scalar theory. Also, it is worth noting that the massless scalar operator ( $\sqrt{-g}\nabla^2$ ) is not conformally invariant, while the LW operator is conformally invariant ( $\sqrt{-g}\Delta_4^2 \rightarrow \sqrt{-\tilde{g}}\tilde{\Delta}_4$ ) under the conformal transformation of  $g_{\mu\nu} \rightarrow e^{2\sigma}\tilde{g}_{\mu\nu}$ .

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## References

- [1] T. D. Lee and G. C. Wick, Nucl. Phys. B **9** (1969) 209.
- [2] T. D. Lee and G. C. Wick, Phys. Rev. D **2**, 1033 (1970).
- [3] Y. F. Cai, T. t. Qiu, R. Brandenberger and X. m. Zhang, Phys. Rev. D **80**, 023511 (2009) [arXiv:0810.4677 [hep-th]].
- [4] B. Grinstein, D. O'Connell and M. B. Wise, Phys. Rev. D **77**, 025012 (2008) [arXiv:0704.1845 [hep-ph]].
- [5] D. Wands, Phys. Rev. D **60**, 023507 (1999) [gr-qc/9809062].
- [6] F. Finelli and R. Brandenberger, Phys. Rev. D **65**, 103522 (2002) [hep-th/0112249].
- [7] D. Baumann, arXiv:0907.5424 [hep-th].
- [8] B. Allen and A. Folacci, Phys. Rev. D **35**, 3771 (1987).
- [9] J. Bros, H. Epstein and U. Moschella, Lett. Math. Phys. **93** (2010) 203 [arXiv:1003.1396 [hep-th]].
- [10] I. Antoniadis, P. O. Mazur and E. Mottola, JCAP **1209** (2012) 024 [arXiv:1103.4164 [gr-qc]].
- [11] A. Youssef, Phys. Lett. B **718** (2013) 1095 [arXiv:1203.3171 [gr-qc]].
- [12] P. D. Mannheim and A. Davidson, Phys. Rev. A **71**, 042110 (2005) [hep-th/0408104].
- [13] F. J. de Urries and J. Julve, J. Phys. A **31** (1998) 6949 [hep-th/9802115].
- [14] T. j. Chen and E. A. Lim, JCAP **1405**, 010 (2014) [arXiv:1311.3189 [hep-th]].
- [15] P. O. Mazur and E. Mottola, Phys. Rev. D **64**, 104022 (2001) [hep-th/0106151].
- [16] E. Mottola, Acta Phys. Polon. B **41**, 2031 (2010) [arXiv:1008.5006 [gr-qc]].
- [17] N. A. Chernikov and E. A. Tagirov, Annales Poincare Phys. Theor. A **9** (1968) 109.

- [18] A. Higuchi and Y. C. Lee, Class. Quant. Grav. **26**, 135019 (2009) [arXiv:0903.3881 [gr-qc]].
- [19] N. Deruelle, M. Sasaki, Y. Sendouda and A. Youssef, JHEP **1209**, 009 (2012) [arXiv:1202.3131 [gr-qc]].
- [20] Y. S. Myung and T. Moon, JCAP **1408**, 061 (2014) [arXiv:1406.4367 [gr-qc]].
- [21] F. F. Faria, Advances in High Energy Physics, vol. 2014, Article ID 520259, 4 pages, 2014 [arXiv:1312.5553 [gr-qc]].
- [22] Y. S. Myung, Phys. Lett. B **730** (2014) 130 [arXiv:1401.1890 [gr-qc]].